FEEDBACK TUTORIAL LETTER

2ND SEMESTER 2020

ASSIGNMENT 1 & 2

Statistics for Economists 2B
SFE612S
Dear Student

Congratulations on the successful completion of your first and second assignments for semester 2 2020.

I am convinced the study guide gave you enough exposure to applications of Statistics for Economists skills in daily financial transactions.

I have no doubts that working through the questions must have in no small way improved on your statistical, analytical and other calculation skills. The attached memoranda are for you to see the step by step methods of realising the final calculations and will also prepare you for the comprehensive test.

Regards,

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Email: dntirampeba@nust.na
Assignment 1

Question 1 [50 marks]

1.1

1.1.1

H₀: \( \mu_1 = \mu_2 = \mu_3 \) \( \checkmark \)

i.e. There is no difference in the three aspects of judgment

H¹: \( \mu_i \neq \mu_j \), for at least one pair \((i, j), i \neq j\) \( \checkmark \)

i.e. At least two aspects of judgment affect the quality of judgement differently.

\[
SS_{TOT} = \sum_{i=1}^{a} \sum_{j=1}^{b} y_{ij}^2 - \frac{\bar{y}^2}{N}
\]

\[
= \left(17^2 + 18.5^2 + \ldots + 24.2^2\right) - \frac{436.5^2}{21}
\]

\[
= 317.34 \checkmark
\]

\[
SS_{treat} = \frac{1}{n_{ij}} \sum_{i=1}^{3} y_{ij}^2 - \frac{\bar{y}^2}{N}
\]

\[
= \frac{1}{7} \times 119^2 + \frac{1}{7} \times 142.8^2 + \frac{1}{7} \times 175^2 - \frac{436.8^2}{21}
\]

\[
= 225.68 \checkmark
\]

\[
SSE = SS_{p} - SS_{treat} \checkmark
\]

\[
= 317.34 - 225.68
\]

\[
= 91.66 \checkmark
\]

\[
MS_{treat} = \frac{SS_{treat}}{k-1} = \frac{225.68}{2} = 112.84 \checkmark
\]

\[
MSE = \frac{SSE}{N-k} = \frac{91.66}{18} = 5.0922 \checkmark
\]

Test statistic:
\[ F = \frac{MS_{\text{treat}}}{MSE} = \frac{112.84}{5.0922} = 22.159 \]

Critical value:
\[ F_{0.05,2,18} = 3.55 \]

Because the test statistic \((F = 22.159)\) is greater than the critical value \((F_{0.05,2,18} = 3.55)\) we reject the null hypothesis.

Conclusion

At 5% significance level, the data provide sufficient evidence to indicate the basis of judgment affects the quality of judgment.

1.1.2

\( H_0 : \mu_i = \mu_j \)

\( H_1 : \mu_i \neq \mu_j\), \( \mu_i \neq \mu_j \) (we compare all possible pairs)

Pair of means \( \mu_i \) and \( \mu_j \) would be declared significantly different if

\[ |\bar{y}_i - \bar{y}_j| > t_{\alpha/n-k}\sqrt{MSE\left(\frac{1}{n_i} + \frac{1}{n_j}\right)} \]

Test statistics: \(|\bar{y}_i - \bar{y}_j|\),

corresponding critical values: \(LSD = t_{\alpha/n-k}\sqrt{MSE\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}\)

Since \(n_1 = n_2 = n_3 = 7\), LSD is constant.

\[
LSD = t_{\alpha/3,36}\sqrt{MSE\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}
= 2.101\sqrt{5.0922\left(\frac{1}{7} + \frac{1}{7}\right)}
= 2.534
\]

| \( |\bar{y}_i - \bar{y}_j| \) | LSD |
|-----------------|-----|
| 3.4             | 2.534 |
| 8               | 2.534 |
Since all values of $|\bar{y}_i - \bar{y}_j|$ are greater than $LSD = 2.534$, we conclude that, at 5% level of significance, all aspects of judgement are different and the best one is the direct experience (i.e lower scores are obtained through this aspect of judgement).

1.2

$H_0 : \mu_1 = \mu_2 = \ldots = \mu_b$

There is no difference in students’ mean performances.

$H_1 : \mu_i \neq \mu_j$ for at least one pair $(i, j)$ and $i \neq j$

At least two two students perform differently from others.

Now, we calculate the test statistics.

$$SS_T = \sum_{i=1}^{a} \sum_{j=1}^{b} \frac{y_{ij}^2}{N} - \frac{\sum y_i^2}{N}$$

$$= (526^2 + 594^2 + \ldots + 420^2) - \frac{18132^2}{18}$$

$$= 65798$$

$$SS_{\text{students}} = \frac{1}{b} \sum_{j=1}^{b} \frac{y_{ij}^2}{N}$$

$$= \frac{1}{3} (1590^2 + 1770^2 + 1374^2 + 1680^2 + 1344^2 + 1308^2) - \frac{18132^2}{18}$$

$$= 63250$$

$$SS_{\text{Aptitude test parts}} = \frac{1}{a} \sum_{j=1}^{a} \frac{y_{ij}^2}{N}$$

$$= \frac{1}{6} (3012^2 + 3090^2 + 2964^2) - \frac{18132^2}{18}$$

$$= 1348$$

$$SSE = SS_T - (SS_{\text{students}} + SS_{\text{Aptitude test parts}})$$

$$= 65798 - (63250 + 1348)$$

$$= 1200$$
F-Statistic for treatment (student performance) comparison:

\[ F = \frac{MS_{\text{students}}}{MSE} = \frac{12650}{120} = 105.4167 \]

F-critical:

\[ F_{0.05,5,10} = 3.33 \]

Because the test statistic \( F = 105.4167 \) is greater than the critical value \( F_{0.05,5,10} = 3.33 \), we reject the null hypothesis.

Conclusion:
At 5% significance level, there is enough evidence that students perform differently in three portions of the SAT.

\[ H_0 : \mu_1 = \mu_2 = \mu_3 \]

There is no difference in trouble given to students by the three portions of SAT.

\[ H_1 : \mu_i \neq \mu_j \text{ for at least one pair } (i, j) \text{ and } i \neq j \]

At least two two portions of SAT give different trouble to students.

F-Statistic for block (three portions of the SAT) comparison:

\[ F = \frac{MS_{\text{block(aptitude)}}}{MSE} = \frac{674}{120} = 5.617 \]
F-critical:

\[ F_{0.05,2,10} = 4.10 \]

Because the test statistic \( F = 5.617 \) is greater than the critical value \( F_{0.05,2,10} = 4.10 \), we reject the null hypothesis.

Conclusion:
At 5% significance level, there is enough evidence to indicate that the three portions of the SAT do not give the students the same trouble.

**Question 2 [50 marks]**

2.1

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>22</th>
<th>17</th>
<th>24</th>
<th>26</th>
<th>28</th>
<th>29</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of passengers</td>
<td>173</td>
<td>149</td>
<td>175</td>
<td>188</td>
<td>186</td>
<td>198</td>
</tr>
</tbody>
</table>

2.1.1 Fit an ordinary least square (OLS) simple linear regression model for predicting number of passengers throughout a particular hour given the temperature at the beginning of the hour.

Let \( x \) be temperature and \( y \) be number of passengers.

\[
SS_{xy} = \sum xy - n\bar{x}\bar{y} = 26071 - 6(24)(178.5) = 367
\]

\[
SS_{xx} = \sum x^2 - n\bar{x}^2 = 3566 - 6(24)^2 = 110
\]

\[
\hat{\beta}_1 = \frac{SS_{xy}}{SS_{xx}} = \frac{367}{110} = 3.3364
\]

\[
\hat{\beta}_0 = \bar{y} - \hat{\beta}_1\bar{x} = 178.5 - 3.336363(24) = 98.4273
\]

Model is \( \hat{y} = 98.4273 + 3.3364x \)

2.1.2 Without using Pearson (product moment) correlation formula, compute coefficient of determination and interpret it.

\[
SS_{yy} = \sum y^2 - n\bar{y}^2 = 192595 - 6(178.5)^2 = 1421.5
\]

\[
r^2 = \frac{SSR}{SST} = \frac{\hat{\beta}_1 SS_{xy}}{SS_{yy}} = \frac{3.336363(367)}{1421.5} = 0.8614
\]

**Meaning:** About 86.14% of total variation in number of passengers (\( y \)) is explained (accounted for) by temperature (\( x \)).

2.1.3 Compute the standard error of the estimate.

\[ \sqrt{ \frac{SSR}{n-2} } \]
\[ SSE = SST - SSR = SS_{y'y} - \hat{\beta}_1 SS_{x'y} = 1421.5 - 3.3364(367) = 197.055 \]

[OR using \( SSE = (1 - r^2)SS_{y'y} = 1421.5(1 - 0.8613756) = 197.055 \)]

Then \( \hat{\sigma} = \sqrt{\frac{SSE}{n-2}} = \sqrt{\frac{197.055}{4}} = 7.019 \)

2.1.4 Construct the 90% prediction interval (P.I.) for number of passengers in an hour with a beginning temperature of 25°C.

\[ \hat{y}\rvert_{25} = 98.427272 + 3.336363(25) = 181.836 \]

Then 90% P.I. for \( \hat{y}\rvert_{25} = \hat{y}\rvert_{25} \pm t_{0.05,4} \hat{\sigma} \sqrt{1 + \frac{1}{n} + \frac{(x - 25)^2}{SS_{xx}}} \)

\[ = 181.836 \pm (2.132)(7.019)\sqrt{1 + \frac{1}{6} + \frac{(24-25)^2}{110}} = 181.836 \pm 16.226 \]

\[ = [165.61, 198.06] = [165, 199] \]

Note: [Rounding appears incorrect but it is correct since these are number of passengers; we round LPL down and UPL up regardless of value of the tenth digit]

2.2

Yes, a linear equation seems to a be good fit for the transformed data √√
log \( y = \log \alpha + \beta \log x \)

Let \( Y = \log y, b_0 = \log \alpha, b = \beta \), and \( X = \log x \). Then the above transformed equation can be rewritten as

\[ Y = b_0 + bX \]

We use the same computational procedure as in 2.1.2 to obtain the coefficient of the regression model.

\[
SS_X = 0.3047 \sqrt{1}, SS_{XY} = 0.1498 \sqrt{1}, \bar{X} = 0.727, \bar{Y} = 0.481
\]

\[ b = \frac{SS_{XY}}{SS_X} = \frac{0.1498}{0.3047} = 0.492 \sqrt{1} \]

\[ b_0 = \bar{Y} - b\bar{X} = 0.481 + 0.492 \times 0.727 = 0.128 \sqrt{1} \]

\[ Y = 0.1283 + 0.492X \sqrt{1} \]

Thus,

\[ 0.1283 = \log \alpha \quad \text{and} \quad b = \beta = 0.492 \sqrt{1} \]

\[ 0.1283 = \log \alpha \Rightarrow \alpha = 10^{0.1283} = 1.344 \sqrt{1} \]

2.3

2.3.1

\[ \hat{x}_i = -18.859 + 16.202x_2 + 0.175x_3 + 11.526x_4 + 13.580x_5 - 5.311x_6 \sqrt{1} \]

2.3.2
\[
AdjR^2 = 1 - \frac{SSE/\sqrt{n-p}}{SST/\sqrt{n-1}} = 1 - \frac{6541.411/21}{95980.352/26} = 0.992
\]

Interpretation: Accounting for degrees of freedom, about 99.2% of the total variation in $\hat{x}_1$ is mutually explained by $x_2, x_3, \ldots, x_6$.

2.3.3

Test for overall adequacy of the fitted model at 5% level?

Hypotheses

$H_0: \beta_2 = \beta_3 = \cdots = \beta_6 = 0$ against $H_a: \beta_i \neq 0$ for at least one $i = 2, 3, \ldots, 6$

Observed F-value

$F_{obs} = 611.590$

Critical F-value

$F_{crit} = F_{0.05}, p-1, n-p = F_{0.05}, 5, 21 = 2.68$

Decision Rule

Reject $H_0$ if $F_{obs} > 2.68$

Conclusion

Since $F_{obs} = 611.590 >> 2.68$, we highly reject $H_0$ and conclude that overall fitted model is significantly adequate at 5% level.

2.3.4

$H_0: \beta_i = 0$ against $H_a: \beta_i \neq 0$ for $i = 2, 3, \ldots, 6$

Since the p-values associated with the regression coefficients are all less than 0.05, we reject the null hypotheses and concluded that all regression coefficient are statistically different from zero.

Therefore, we would not consider eliminating any variable.
Assignment 2

Question 1 [35 marks]

1.1 [10]

Step 1:

The null and alternative hypotheses are

\[ H_0 : p_1 = 0.12, p_2 = 0.29, p_3 = 0.11, p_4 = 0.10, p_5 = 0.14, p_6 = 0.24 \]

\[ H_1 : \text{At least one of the proportions is different from stated proportion.} \]

Step 2

The significance level is 0.01

Step 3: Computation of the test statistic

In step 3, we have to compute the test statistic using the formula

\[ \chi^2 = \sum \frac{f_o^2}{f_e} - n \]

From this formula, we notice that we must find the expected frequencies \( f_e \) as they are not given.
We recall that $f_e = np_i$

Thus, the expected frequency for the first category is given by $f_1 = 519 \times 0.12 = 62.28$, the expected frequency for the second category is given by $f_2 = 519 \times 0.29 = 150.51$

the expected frequency for the third category is given by $f_3 = 519 \times 0.11 = 57.09$, the expected frequency for the first category is given by $f_4 = 519 \times 0.1 = 51.9$, the expected frequency for the fifth category is given by $f_5 = 519 \times 0.14 = 72.66$, and the expected frequency sixth category is given by $f_6 = 519 \times 0.24 = 125.6$

$$\chi^2_{calc} = \sum \frac{f_o^2}{f_e} - n$$

$$\chi^2_{calc} = \frac{88^2}{62.28} + \frac{135^2}{150.51} + \frac{52^2}{57.09} + \frac{40^2}{51.9} + \frac{76^2}{72.66} + \frac{128^2}{125.6} - 519 \sqrt{\chi^2} = 15.659 \sqrt{15.659}$$

Step 4: Select the critical value: $\chi^2_{a,k-1}$

There are six age categories. Hence, $k = 6$ and the number of degrees of freedom is $k - 1 = 6 - 1 = 5$. The critical value is $\chi^2_{0.015} = 15.08627$.

Step 5: Formulation of the rejection rule

We reject $H_0$ for a value of statistic greater than $\chi^2_{0.015} = 15.08627$. In our case, $\chi^2_{calc} = 15.609$ is greater than $\chi^2_{0.015} = 15.08627$. Hence, $H_0$ is rejected.

Step 6: conclusion

At 1% significance level, we conclude that the data provide enough evidence to indicate that the distribution of customer ages in the Snoop report does not agree with that of the sample report.

1.2

H0: The performance and the size of a company are statistically independent.

H1: The performance and the size of a company are associated

Computation of expected values ($E_i$) and test statistic ($D^2$):
\[ E_i = \frac{\text{Column total} \times \text{row total}}{\text{Grandtotal}} \]

\[ D^2 = \sum \frac{O_i^2}{E_i} - n \]

<table>
<thead>
<tr>
<th>Company size</th>
<th>Performance(% price change)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>&lt;0%</td>
</tr>
<tr>
<td>Small</td>
<td>66(77.0)</td>
</tr>
<tr>
<td>Medium</td>
<td>24(29.3)</td>
</tr>
<tr>
<td>Large</td>
<td>55(38.7)</td>
</tr>
<tr>
<td>Total</td>
<td>145</td>
</tr>
</tbody>
</table>
\[ D^2 = \sum\frac{O_i^2}{E_i} - n \]
\[ = \frac{66^2}{77} + \frac{90^2}{85} + \ldots + \frac{10^2}{16} - 420 \]
\[ = 19.09 \]  
\[ X_{6}^{2(0.05)} = 12.592 \]

Conclusion:

Because \( D^2 = 19.09 > X_{6}^{2(0.05)} = 12.592 \) we reject Ho.√

Thus our financial analyst has demonstrated that there is significant relationship between the performance of firm and its size. √

1.3

Test Hypotheses

\( H_0: \ p_i = 1/6 \) for all \( i = 1, 2, \ldots, 6 \) i.e. The die is balanced(fair)

\( H_a: \) At least one \( p_i \neq \frac{1}{6} \) for some \( i = 1, 2, 3, 4 \) i.e. The die is not balanced (fair)

Since we are testing adequacy of a discrete uniform model, \( X \sim Uni(n = 300) \), the expected counts are simply \( E_i = NP_i = 300(1/6) = 50 \) for all \( i = 1, 2, \ldots, 6 \). Then the observed and expected counts are as follows.

\[
\begin{array}{c|c|c}
\text{Score} & O_i & E_i = NP_i \\
\hline
1 & 62 & 50 \\
2 & 45 & 50 \\
3 & 63 & 50 \\
4 & 32 & 50 \\
5 & 47 & 50 \\
\end{array}
\]
(Note: This table is optional and it has no mark at all. However, if a candidate only gives this table without explanation just above then will still be awarded the same 2 marks above.)

**Observed $\chi^2$-value**

$$\chi^2_{\text{obs}} = \sum_{i=1}^{6} \frac{O_i^2}{E_i} - 80 = \left[ \frac{62^2}{50} + \cdots + \frac{51^2}{50} \right] - 300 = \boxed{313.44} - 300 = 13.44$$

**Critical $\chi^2$-value**

$$\chi^2_{\text{crit}} = \chi^2_{\alpha, k-p-1} = \chi^2_{0.010,5} = 15.08627$$

**Decision Rule**

Reject $H_0$ if $\chi^2_{\text{obs}} > \chi^2_{\text{crit}} = 15.08627$

**Decision Making**

Since $\chi^2_{\text{obs}} = 13.44 < \chi^2_{\text{crit}} = 15.08627$, we do not reject $H_0$.

**Conclusion**

At 1% significance level, we conclude that the data do not provide sufficient evidence to infer that the die is not balanced (fair).

---

**Question 2 [44 marks]**

2.1

<table>
<thead>
<tr>
<th>Year</th>
<th>Quarter</th>
<th>Quarter rate</th>
<th>4_MA(8marks)</th>
<th>Seasonal ratio (4 marks)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006</td>
<td>1</td>
<td>0.561</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.702</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.8</td>
<td>0.6595</td>
<td>121.304</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.568</td>
<td>0.66575</td>
<td>85.31731</td>
</tr>
<tr>
<td>2007</td>
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<td>0.575</td>
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<td>84.71455</td>
</tr>
<tr>
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</tr>
<tr>
<td></td>
<td>3</td>
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</tr>
<tr>
<td></td>
<td>4</td>
<td>0.605</td>
<td>0.70125</td>
<td>86.27451</td>
</tr>
<tr>
<td>2008</td>
<td>1</td>
<td>0.594</td>
<td>0.683875</td>
<td>86.85798</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.738</td>
<td>0.665875</td>
<td>110.8316</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.729</td>
<td>0.66875</td>
<td>109.0093</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.6</td>
<td>0.6685</td>
<td>89.75318</td>
</tr>
</tbody>
</table>
### Seasonal Median and Index Calculation

#### Seasonal Median

<table>
<thead>
<tr>
<th>Year</th>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006</td>
<td>84.71455</td>
<td>106.6667</td>
<td>121.304</td>
<td></td>
<td>85.31731</td>
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<td>86.85798</td>
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<td>2009</td>
<td>89.51708</td>
<td>110.4497</td>
<td></td>
<td></td>
<td>87.94573</td>
</tr>
<tr>
<td>2010</td>
<td>88.18753</td>
<td>108.5582</td>
<td>118.4401</td>
<td>87.11012</td>
<td>402.296</td>
</tr>
</tbody>
</table>

#### Seasonal Index

The seasonal index is calculated as the ratio of the median of each quarter to the overall median. The adjustment factor is then used to calculate the adjusted seasonal index.

#### Adjusted Adjustment Factor

\[
\text{Adjustment factor} = \frac{k \times 100}{\sum \text{Median seasonal index}} = \frac{400}{402.296} = 0.994
\]

#### Adjusted Seasonal Index

<table>
<thead>
<tr>
<th>Quarter</th>
<th>Seasonal Median (SM)</th>
<th>Adjusted Seasonal Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1</td>
<td>88.18753</td>
<td>87.68422</td>
</tr>
<tr>
<td>Q2</td>
<td>108.5582</td>
<td>107.9386</td>
</tr>
<tr>
<td>Q3</td>
<td>118.4401</td>
<td>117.7642</td>
</tr>
<tr>
<td>Q4</td>
<td>87.11012</td>
<td>86.61296</td>
</tr>
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</table>
### 2.3

<table>
<thead>
<tr>
<th>Year</th>
<th>Quarter</th>
<th>rate</th>
<th>SI</th>
<th>De-seasonalised index</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006</td>
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<td>4</td>
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<td>0.655791</td>
</tr>
<tr>
<td>2007</td>
<td>1</td>
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<td>86.61296</td>
<td>0.773556</td>
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</table>

Interpretation of the first de-seasonalised index (0.6398):  
2006 quarter 1 occupancy rate would have been higher at 0.6398, instead of the actual sales of 0.561, had seasonal influences not been present. ✔️

2.4 [10]
Trend Line formula: \( y = bx + a \)

Slope \( b = \frac{\sum xy}{\sum x^2} = \frac{6.977}{2660} = 0.00263 \)

Intercept \( a = \frac{\sum y}{n} = \frac{13.889}{20} = 0.6945 \)

Trend Line: \( y = 0.00263x + 0.6945 \)
Estimate the trend value of the time series for Quarter 3 in 2011.

Quarter 3 in 2011: \( T = \hat{y} = 0.00263(25) + 0.6945 = 0.76 \)

2.6 [4]

The seasonally-adjusted trend estimate is calculated using \( T \times S \).

where the \( S \) is the seasonal index for the specific period (Q3)

Thus, \( T \times S = 0.76 \times \frac{117.7642}{100} \sqrt{\sqrt{\cdot}} = 0.895 \sqrt{\cdot} \).

**Question 3 [6 marks]**

<table>
<thead>
<tr>
<th>Year</th>
<th>Quarter</th>
<th>Sales</th>
<th>Exponentially smoothed sales (( w = 0.1 ))</th>
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<tr>
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**Question 4 [15 marks]**

4.1.

Price relative for food = \( \frac{p_1}{p_0} \times 100\% = \frac{45}{40} \times 100\% = 112.5\% \)

This shows that since 1981, the price of food has increased by 12.5\%. √

Price relative for accommodation = \( \frac{p_1}{p_0} \times 100\% = \frac{40}{30} \times 100\% = 133.33\% \)

This shows that the average price of accommodation has increased by 33.33\% since 1981. √
Price relative for petrol = \( \frac{p_1}{p_0} \times 100\% = \frac{1.40}{1.30} \times 100\% = 107.69\% \)

This shows that the average price of petrol has increased by 7.69\% since 1981. √

4.2

Quantity relative for food = \( \frac{p_1}{p_0} \times 100\% = \frac{25}{30} \times 100\% = 83.33\% \)

This shows that since 1981, the food consumed by typical family of four has decreased by 16.67\%. √

Quantity relative for accommodation = \( \frac{p_1}{p_0} \times 100\% = \frac{5}{8} \times 100\% = 62.5\% \)

This shows that the consumption of accommodation units has decreased by 37.5\% since 1981. √

Quantity relative for petrol = \( \frac{p_1}{p_0} \times 100\% = 75 \times 100\% = 80.00\% \)

This shows that the number of units of petrol consumed by a typical family of four has decreased by 20\% since 1981. √

4.3-4.5

<table>
<thead>
<tr>
<th>Item</th>
<th>Quantities</th>
<th>Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1981 (q_0)</td>
<td>1982 (q_1)</td>
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<td>Food</td>
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<td>25</td>
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<tr>
<td>Accommodation</td>
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<td>5</td>
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<td>Petrol</td>
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<td>Entertainment</td>
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<td>8</td>
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<tr>
<td>Total</td>
<td>(\sum p_0 = 318)</td>
<td>(\sum p_0 = 318)</td>
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</tbody>
</table>
4.3.

Laspeyres price index = \[ \frac{\sum (p_1 q_0)}{\sum p_0 q_0} \times 100\% = \frac{1895}{1637.5} \times 100\% = 115.725\% \]

If quantities (number of units consumed) are held constant at 1981 (base period) levels, the price of the number of units consumed by a typical family has increased by, on average 15.725\% since 1981.

[4]

4.4

Paasche’s quantity index = \[ \frac{\sum (p_1 q_1)}{\sum p_1 q_0} \times 100\% = \frac{1505}{1895} \times 100\% = 79.42\% \]

If prices are held constant at current period levels (1982), the quantities (the number of units consumed by a typical family) would have increased by 20.58\%. \[4\]

4.5

unweighted composite (aggregate) price index = \[ \frac{\sum p_1}{\sum p_0} \times 100\% = \frac{98.4}{81.8} \times 100\% = 121.03\% \]

Total: 100