FEEDBACK TUTORIAL LETTER

1st SEMESTER 2019

ASSESSMENT 2

FOR

STATISTICS FOR ECONOMISTS 2A
SFE611S
Dear Student

Congratulations on the successful completion of your second assignment for semester 1 2019.
I am convinced the study guide gave you enough exposure to applications of Statistics for Economists skills in daily financial transactions.
I have no doubts that working through the questions must have in no small way improved on your statistical, analytical and other calculation skills.
I wish to point out some of the changes made. For question 2.5, some information was not provided to be able to answer the question. As a result, I have opted to remove this question from the assignment. Consequently, the total mark has changed.
The attached memorandum is for you to see the step by step methods of realising the final calculations and will also prepare you towards the end of June examinations.

Regards,

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ASSIGNMENT 2

Question 1 [20 marks]

1.1

1.1.1.

\[ \bar{X} \pm t_{s,n-1} \times \frac{s}{\sqrt{n}} \]

\[ 83.75 \pm 2.093 \sqrt{\frac{28.966}{20}} \]

\[ (70.194 \sqrt{ \ ; 97.306 \sqrt{ } } ) \]

1.1.2. We are 95% sure that the true population average cost of a sleeping bag cost lies between 70.194 and 97.306 \[ \sqrt{} \]

1.2

\[ \bar{X} \pm \frac{Z_{\alpha}}{2} \frac{\sigma}{\sqrt{n}} \]

\[ 79.3 \pm 1.96 \sqrt{\frac{7.8}{\sqrt{32}}} \]

\[ (76.597 \sqrt{ \ ; 82.003 \sqrt{ } } ) \]

1.3

\[ 10 = Z_{0.02} \times \frac{40}{\sqrt{n}} \]

\[ 10 = 2.05 \times \frac{40}{\sqrt{n}} \]

\[ n = \left( \frac{2.05 \times 40}{10} \right)^2 \sqrt{ } = 67.24 \approx 68 \sqrt{ } \]

1.4
\[ \hat{p} = \frac{114}{200} = 0.57, \hat{q} = 0.43, \] the confidence interval is thus
\[ \hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p} \hat{q}}{n}} \]
\[ 0.57 \pm 2.17 \sqrt{\frac{0.57 \times 0.43}{200}} \]
\[ (0.494 \sqrt{; \ 0.646 \sqrt{)} \] \[ \text{[3]} \]

1.5

1.5.1

\[ S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2} \]
\[ = \frac{12 \times 4^2 + 18 \times 6^2}{12 + 18 - 2} = 30 \sqrt{ \]
\[ X_1 - X_2 \pm t_{n_1+n_2-2} \times S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \]
\[ 84 - 77 \pm t_{0.005,18} \times \sqrt{30 \left( \frac{1}{12} + \frac{1}{18} \right)} \]
\[ 84 - 77 \pm 2.763 \sqrt{30 \left( \frac{1}{12} + \frac{1}{18} \right)} \]
\[ (1.3601 \sqrt{; \ 12.64 \sqrt{)} \] \[ \text{[4]} \]

Because the confidence interval does not contain zero, the mean difference is statistically significant different from zero. \[ \text{[2]} \]
Question 2 [43 marks]

2.1 [7]

\[ H_0 : \mu = 162.5 \]
\[ H_1 : \mu \neq 162.5 \]

Test statistic:

\[ z = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \sim N(0,1) \]
\[ = \frac{165.2 - 162.5}{6.9/\sqrt{50}} \]
\[ = 2.77 \]

P-value: 0.0056

Since the p-value is small (less 0.05), we reject the null hypothesis.

Conclusion:

We conclude that there is enough evidence to believe that there has been a change in the average height.

2.2 [8]

\[ H_0 : \mu \leq 20000 \]
\[ H_1 : \mu > 20000 \]

Test statistic:

\[ z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \sim N(0,1) \]
\[ = \frac{23500 - 20000}{3900/\sqrt{100}} \]
\[ = 8.97 \]

Critical value:
$Z_{0.05} = 1.645$ √

Since the test statistic of 8.97 is greater than the critical value of 1.645, we reject the null hypothesis. √

Conclusion:

We conclude that there is enough evidence to indicate that automobiles are driven on average more than 20,000 kilometers per year. √√

P-value is less 0.0002 or less than 0.002 √

2.3 [7]

$H_0 : \pi = 0.40$ √

$H_1 : \pi \neq 0.40$ √

$\hat{p} = \frac{9}{30} = 0.3, \hat{q} = 0.7$ √

Test statistic:

$z = \frac{\hat{p} - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} \sim N(0,1)$

$= \frac{0.3 - 0.40}{\sqrt{\frac{0.40(0.60)}{20}}} \sqrt{\frac{0.40(0.60)}{20}}$

$= -1.118$ √

Critical value:

$-Z_{0.025} = -1.96$ √

Since the test statistic of -1.118 is greater than the critical value of -1.96, we do not reject the null hypothesis. √
Conclusion:

We conclude that there is no enough evidence to indicate that the population proportion of pasta lovers is different from 40% \(\sqrt{\sqrt{}}\)

P-value: 0.2628 \(\sqrt{\sqrt{}}\)

2.4

\[ H_0 : \pi_1 - \pi_2 \leq 0 \sqrt{\sqrt{}} \]
\[ H_1 : \pi_1 - \pi_2 > 0 \sqrt{\sqrt{}} \]

Step 2: Compute the test statistic

Test statistic:

\[
Z = \frac{(\hat{p}_1 - \hat{p}_2) - D_0}{\sigma_{\hat{p}_1 - \hat{p}_2}} = \frac{(\hat{p}_1 - \hat{p}_2) - D_0}{\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}} \sim N(0,1)
\]

\[
Z = \frac{(0.60 - 0.48) - 0}{\sqrt{\frac{(0.60)(0.4)}{200} + \frac{(0.48)(0.52)}{500}}} = 2.911 \sqrt{\sqrt{}}
\]

Step 3: Determine the critical values

\[ Z_{0.05} = 1.645 \sqrt{\sqrt{}} \]

Step 4: Decision

We reject the null hypothesis \(H_0\) since \(Z = 2.911\) is greater than the critical value 1.645 \(\sqrt{\sqrt{}}\)

Conclusion: At 5% significance level, there is sufficient evidence to indicate that the population proportion of town voters is higher than that in county. \(\sqrt{\sqrt{}}\)

2.5

Hypotheses

\[ H_0 : \mu_1 - \mu_2 = 0 \sqrt{\sqrt{}} \]
\[ H_1 : \mu_1 - \mu_2 \neq 0 \sqrt{\sqrt{}} \]

Test statistic
\[ S^2_p = \frac{(n_1 - 1)S^2_1 + (n_2 - 1)S^2_2}{n_1 + n_2 - 2} \]
\[ = \frac{9 \times 2.79^2 + 9 \times 2.46^2}{10 + 10 - 2} = 6.918\sqrt{\ldots} \]

Test statistic:

\[ t = \frac{\bar{x} - \bar{y} - D_0}{\sigma_{(\bar{x} - \bar{y})}} = \frac{\bar{x} - \bar{y} - D_0}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \]
\[ t = \frac{(22.15 - 20.6) - 0}{\sqrt{6.918 \sqrt{\frac{1}{10} + \frac{1}{10}}} = 1.318\sqrt{\ldots} } \]

The hypothesis is a two-sided test so we use \( t_{18,0.025} = 2.101\sqrt{\ldots} \)

We do not reject the null hypothesis \( H_0 \) since \( t = 1.318 \) is not greater than the critical value \( 2.101\sqrt{\ldots} \)

Conclusion

At 5% significance level, there is insufficient evidence to indicate that the means of the two hedge funds’ performances are different \( \sqrt{\ldots} \)

2.6

\[ 100(1 - \alpha)\% \text{ confidence interval for } \sigma^2 \text{ is } \frac{(n-1)S^2}{\chi^2_{\alpha/2,n-1}} < \sigma^2 < \frac{(n-1)S^2}{\chi^2_{1-\alpha/2,n-1}} \]
\[ \frac{4 \times 0.815}{\chi^2_{0.025,4}} < \sigma^2 < \frac{4 \times 0.815}{\chi^2_{0.975,4}} \]
\[ \frac{4 \times 0.815}{11.14329} < \sigma^2 < \frac{4 \times 0.815}{0.48442} \]
\[ 0.293\sqrt{\ldots}, 6.73\sqrt{\ldots} \]
Steps:

\[ H_0 : \sigma^2 = 1 \]

\[ H_1 : \sigma^2 \neq 1 \text{ (two-tailed test)} \]

Test statistic:

\[ \chi^2 = \frac{(n-1)S^2}{\sigma_o^2} = \frac{4 \times 0.815}{1} = 3.26 \]

Two-tailed test

\[ \chi^2 > \chi^2_{\frac{\alpha}{2}, n-1} = 11.14329 \]

or

\[ \chi^2 < \chi^2_{\frac{1-\alpha}{2}, n-1} = 0.48442 \]

We do not reject Ho as neither condition is met.

Conclusion:

There is no enough evidence to indicate that the variance is different from one. Hence, the manufacturer’s claim that \( \sigma^2 = 1 \) is valid at 5% significance level.

Question 3 [12 marks]

3.1. [10]

\( H_0 \): The three population distributions are identical.

\( H_1 \): At least two of the three population distributions differ in location.

Ranks for Calculators

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\[ H = \frac{12}{n(n+1)} \sum_{i=1}^{k} T_i^2 - 3(n+1) \]
\[ = \frac{12}{18(19)} \left[ \frac{25^2}{5} + \frac{56^2}{7} + \frac{90^2}{6} \right] - 3(19) \sqrt{\ } \]
\[ = 10.47 \sqrt{\ } \]

Critical value: \( X^2_{0.05,2} = 5.99147 \sqrt{\ } \)

One mark for correct decision rule \( \sqrt{\ } \)

Because \( H = 10.47 > X^2_{0.05,2} = 5.99147 \), we reject the null hypothesis stating that the three population distributions are identical. \( \sqrt{\ } \)

Therefore, at \( \alpha =0.05 \) level of significance, there is evidence to conclude the operating times for three types of scientific pocket calculators before a recharge is required differ in location. \( \sqrt{\ } \)

3.2 \( 0.005 < P-value < 0.01 \) \( \sqrt{\ } \) [2]