FEEDBACK TUTORIAL LETTER

1st SEMESTER 2019

ASSIGNMENT 1

STATISTICS FOR ECONOMISTS 2A
SFE611S
Dear Student,

Congratulations on the successful completion of your first assignment for semester 1 2019. I am convinced the study guide gave you enough exposure to applications of Statistics for Economists skills in daily financial transactions. I have no doubts that working through the questions must have in no small way improved on your statistical, analytical and other calculation skills.

I wish to point out some of the changes made. For question 1.2, the table that was supposed to be used for computation of coefficient of determination was misplaced (i.e., place in 1.1.4). This has resulted to confusion and some of you could not answer it. As a result, I have opted to remove this question from the assignment. Consequently, the total mark has changed to 17 marks. For question 2, there was error in the total mark, the total for this question is 21 marks instead of 25 marks. The overall total for this assignment has changed to 88 marks due to the above-mentioned changes.

The attached memorandum is for you to see the step by step methods of realising the final calculations and will also prepare you towards the end of June examinations.

Regards,

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ASSIGNMENT 1

Question 1 [17 marks]

1.1.  
1.1.1.  
\[
\bar{X} = \frac{1}{n} \sum X_i = \frac{86.92}{30} = 2.897 \sqrt{ } \\
\text{On average, a staff salary is 2.897 dollars per pupil. \checkmark}
\]

1.1.2.  
\[
\text{Median} = \frac{2.83 + 2.89}{2} = 2.86 \sqrt{ } .
\]

1.1.2.  
\[
\text{Of the 30 staff selected, 50\% of them, earns less than 2.86 dollars per pupil and the other 50\% of staff earns more than 2.86 dollars per pupil. \checkmark}
\]

1.1.3.  
Mode: 2.52 \sqrt{ } .

2.52 was the most repeated salary \checkmark.  

1.1.4.  
First quartile: \[ Q_1 \text{ position} = 0.25(n + 1) = 0.25(31) = 7.75 \checkmark \]
\[
Q_1 \text{ value} = 2.45 + 0.75 \times (2.52 - 2.45) = 2.50 \checkmark
\]

Third quartile: \[ Q_3 \text{ position} = 0.75(n + 1) = 0.75(31) = 23.25 \checkmark \]
\[
Q_3 \text{ value} = 3.36 + 0.25 \times (3.37 - 3.36) = 3.36 \checkmark
\]

1.1.5.  
Quartile deviation \[ = \frac{3.36 - 2.50}{2} = 0.43 \sqrt{ } \]

1.1.6.  
\[
\sigma^2 = \frac{\sum x^2 - n\bar{x}^2}{n - 1} = \frac{260.34 - 30(2.897)^2}{29} = 0.293 \sqrt{ } \checkmark
\]
\[ CV = \frac{S}{\bar{x}} \times 100\% \]

\[ = \sqrt{\frac{0.293}{2.897}} \times 100\% = 18.69\% \]  

1.2

<table>
<thead>
<tr>
<th>Interval</th>
<th>Freq ((f_i))</th>
<th>(x_i)</th>
<th>(x_if_i)</th>
<th>(x_i^2f_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.6-12.5</td>
<td>15</td>
<td>10.55</td>
<td>158.25</td>
<td>1669.538</td>
</tr>
<tr>
<td>12.6-16.5</td>
<td>20</td>
<td>14.55</td>
<td>291</td>
<td>4234.05</td>
</tr>
<tr>
<td>16.6-20.5</td>
<td>5</td>
<td>18.55</td>
<td>92.75</td>
<td>1720.513</td>
</tr>
<tr>
<td>20.6-24.5</td>
<td>7</td>
<td>22.55</td>
<td>157.85</td>
<td>3559.518</td>
</tr>
<tr>
<td>24.6-28.5</td>
<td>3</td>
<td>26.55</td>
<td>79.65</td>
<td>2114.708</td>
</tr>
<tr>
<td>Tot</td>
<td>(\Sigma f_i = 50)</td>
<td>(\Sigma x_i f_i = 779.5)</td>
<td>(\Sigma x_i^2 f_i = 13298.33)</td>
<td></td>
</tr>
</tbody>
</table>

\[ \bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{779.50}{50} = 15.59 \]  

\[ S^2 = \frac{\sum f_i x_i^2 \ - \ n \bar{x}^2}{n - 1} = \]  

\[ = \frac{13298.33 \ - \ 50(15.59)^2}{49} = 23.386 \]

\[ CV = \frac{S}{\bar{x}} \times 100\% \]

\[ = \frac{\sqrt{23.386}}{15.59} \times 100\% = 31.02\% \]
Question 2 [21 marks]

2.1.

2.1.1. Measurement level: nominal [2]

2.1.2. 32% [2]

2.1.3. \(0.32 + 0.25 + 0.30 = 0.87\) [2]

2.1.4. \(0.05 + 0.06 + 0.02 = 0.13\) [4]

2.1.5. \(0.06 + 0.25 + 0.02 + 0.30 = 0.63\) [3]

2.1.6. \(P(E \text{ or } G|H) = P(E|H) + P(G|H)\) [3]

\[0.25 + 0.30 + 0.30 + 0.87 = 0.63\]

2.2. A: Purchase latex, \(\bar{A}\): purchase semigloss, B: Purchase roller √

\[P(A) = 0.75, \ P(\bar{A}) = 0.25, \ P(B|A) = 0.60, \ P(B|\bar{A}) = 0.30 \ \checkmark\]

\[P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|\bar{A})P(\bar{A})} = \frac{0.6 \times 0.75}{0.6 \times 0.75 + 0.3 \times 0.25} = 0.857 \checkmark\]

2.3. \(B_1\): Product from machine 1, \(B_2\): Product from machine 2, and \(B_3\): Product from machine 3 √

\[A: \text{ Product is defective} \ \checkmark\]

\[P(B_1) = 0.30, \ P(B_2) = 0.45, \ P(B_3) = 0.25\]

\[P(A|B_1) = 0.02, \ P(A|B_2) = 0.03, \ P(A|B_3) = 0.02 \ \checkmark\]

\[P(A) = \sum P(A|B_i)P(B_i) = 0.02 \times 0.3 + 0.03 \times 0.45 + 0.02 \times 0.25 = 0.0245 \checkmark\]

Question 3 [37 marks]

3.1.

<table>
<thead>
<tr>
<th>(x)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Tot</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f(x))</td>
<td>0.41</td>
<td>0.37</td>
<td>0.16</td>
<td>0.05</td>
<td>0.01</td>
<td>1</td>
</tr>
<tr>
<td>(xf(x))</td>
<td>0</td>
<td>0.37</td>
<td>0.32</td>
<td>0.15</td>
<td>0.04</td>
<td>0.88</td>
</tr>
<tr>
<td>(x^2f(x))</td>
<td>0</td>
<td>0.37</td>
<td>0.64</td>
<td>0.45</td>
<td>0.16</td>
<td>1.62</td>
</tr>
</tbody>
</table>

3.1.1.
\[ E(X) = \sum x f(x) = 0 \times f(0) + \ldots + 4 \times f(4) \]
\[ = 0 \times 0.41 + \ldots + 4 \times 0.01 = 0.88 \sqrt{} \]
\[ Var(X) = \sum x^2 f(x) - (E(X))^2 = 0^2 \times 0.41 + \ldots + 4^2 \times 0.01 - 0.88^2 \]
\[ = 1.62 - 0.88^2 = 0.8456 \sqrt{} \]
\[ \sigma = \sqrt{0.8456} = 0.9196 \sqrt{} \]

3.1.2.
\[ E(X^2 + 3) = E(X^2) + 3 = 1.62 + 3 = 4.62 \sqrt{} \sqrt{} \sqrt{} \]

3.2.
Let \( Y = X^2 + 3 \)
\[ Var(X^2 + 3) = Var(Y) \]
\[ = E[(Y - \mu_Y)^2] \]
\[ = E[(X^2 + 3 - 4.62)^2] \]
\[ = E[(X^2 - 1.62)^2] \]
\[ = E[X^4 - 3.24X^2 + 2.6244 \sqrt{}] \]
\[ = E[X^4] - 3.24E(X^2) + 2.6244 \sqrt{}, \]
where \( E[X^4] = \sum x^4 f(x) = 0^4 \times 0.41 + \ldots + 4^4 \times 0.01 = 9.54 \sqrt{} \)
\[ Var(X^2 + 3) = 9.54 - 3.24 \times 1.62 + 2.6244 = 6.9156 \sqrt{} \]

3.2.1.
\[ \int_1^\infty k x^{-3} dx = 1 \]
\[ k \left( \frac{x^{-3}}{-3} \right) \bigg|_1^\infty = 1 \]
\[ -k \frac{1}{3} (0 - 1) \sqrt{} = 1 \]
\[ k \frac{1}{3} = 1 \]
\[ \therefore k = 3 \sqrt{} \]
3.2.2.

3.2.2.1.

\[ E(X) = 3 \int_1^\infty x^{-3} \, dx = 3 \left[ \frac{x^{-2}}{-2} \right]_1^\infty = -\frac{3}{2} \sqrt{2} \]

\[ 3.2.2.2. \]

\[ Var(X) = E(X^2) - (E(X))^2 \]

\[ E(X^2) = 3 \int_1^\infty x^{-2} \, dx = -3 \left[ x^{-1} \right]_1^\infty = 3 \]

\[ Var(X) = 3 - \frac{9}{4} = \frac{3}{4} \]

\[ 3.2.2.3. \]

\[ P(X > 4) \sqrt[4]{4} = 3 \int_4^\infty x^{-4} \, dx \]

\[ = 3 \left[ \frac{x^{-3}}{-3} \right]_4^\infty \]

\[ = \frac{1}{4^3} \sqrt{4} = \frac{1}{64} \]

\[ 3.3 \]

3.3.1

\[ P(X > 8) = P \left( z > \frac{224 - 200}{15} = 1.6 \right) \]

\[ = 0.0548 \]

\[ 3.3.2 \]

\[ P(X < X^*) = 0.25 \]

\[ \therefore P(z < z^*) = 0.25 \Rightarrow z^* = \frac{X^* - 200}{15} = -0.67 \Rightarrow X^* = -0.67 \times 15 + 200 = 189.95 \]
3.4.

\[
P(X < 3) = \int_{0}^{3} \frac{1}{4} e^{-\frac{x}{4}} dx dx
\]

\[
= -e^{-\frac{1}{2}} \bigg|_{0}^{3}
\]

\[
= - \left( e^{-\frac{3}{4}} - e^{-0} \right)
\]

\[
= 1 - e^{-\frac{3}{4}} = 0.528 \sqrt{\ }
\]

Y follows a binomial distribution with n=6, p=0.528

\[
p(4) + p(5) + p(6)
\]

\[
= 6C4 \times 0.528^4 \times (1 - 0.528)^2 + 6C5 \times 0.528^5 \times (1 - 0.528)^1
\]

\[
+ 6C6 \times 0.528^6 \times (1 - 0.528)^0 = 0.39761
\]

\[
E(X) = 14
\]

3.5.

\[
E(X) = 14
\]

3.6.

\[
P(X \leq 3) = 1 - p(4) + p(5) = 1 - [5C4 \times 0.75^4 \times (1-0.75)^1 + 5C5 \times 0.75^5 \times (1-0.75)^0] = 0.3672
\]
Question 4 [13 marks]

4.1.
\[ E(X) = \sum x p(x) \]
\[ = 2 \times 0.4 + 4 \times 0.6 \]
\[ = 3.2 \sqrt{ } \]  

4.2.
\[ E(Y) = \sum y p(y) \]
\[ = 1 \times 0.25 + 3 \times 0.5 + 5 \times 0.25 \]
\[ = 3 \sqrt{ } \]  

4.3.
\[ E(XY) = \sum \sum xy p(x, y) \]
\[ = 1 \times 2 \times 0.1 + ... + 5 \times 4 \times 0.15 \]
\[ = 9.6 \sqrt{ } \]  

4.4.
\[ Var(X) = \sum x^2 p(x) - \mu_X^2 \]
\[ = 2^2 \times 0.4 + 4^2 \times 0.6 - (3.2)^2 \sqrt{ } \]
\[ = 0.96 \sqrt{ } \]  

4.5.
\[ Var(Y) = \sum y^2 p(y) - \mu_Y^2 \]
\[ = 1^2 \times 0.25 + 3^2 \times 0.5 + 5^2 \times 0.25 - (3)^2 \sqrt{ } \]
\[ = 2 \sqrt{ } \]  

4.6.
\[ Cov(X, Y) = E(XY) - E(X) \times E(Y) \]
\[ = 9.6 - 3.2 \times 3 \]
\[ = 0 \sqrt{ } \]  

4.7
\[ \rho = 0 \sqrt{ } \]  

There is no linear relationship between X and Y.
4.7. 
\[
P(Y | X = 2) = \frac{P(X, Y)}{P(X = 2)}
\]
\[
\begin{align*}
0.1 & = 0.25, x = 1 \\
0.4 & \\
0.2 & = 0.5, x = 3 \\
0.4 & \\
0.1 & = 0.25, x = 5 \\
0.4 & \\
0, & \text{ elsewhere}
\end{align*}
\]

[4]

END OF FEED-BACK TUTORIAL LETTER