4. Understanding Interest Rates
Present Value

• A dollar paid to you one year from now is less valuable than a dollar paid to you today
Discounting the Future Value

Let $i = .10$

In one year  $\$100 \times (1 + 0.10) = \$110$

In two years  $\$110 \times (1 + 0.10) = \$121$

or  $100 \times (1 + 0.10)^2$

In three years  $\$121 \times (1 + 0.10) = \$133$

or  $100 \times (1 + 0.10)^3$

In $n$ years

$\$100 \times (1 + i)^n$
Discounting the Future Value

• Formula for calculating future value
• $FV = P \ (1+r)^n$
• We can also work backward from future amounts to present.

• For example $121 in two years from now is worth $100 today as below:

$$100 = \frac{$121}{(1+0.10)^2}$$

• The process of calculating today’s value from money received in the future, is called discounting the future.
Simple Present Value

\[ PV = \text{today's (present) value} \]

\[ CF = \text{future cash flow (payment)} \]

\[ i = \text{the interest rate} \]

\[ PV = \frac{CF}{(1 + i)^n} \]
Four Types of Credit Market Instruments

- **Simple Loan**: The lender provides the borrower with an amount of funds, which must be repaid to the lender at the maturity date plus interest payments.
  - E.g. commercial loan to business

- **Fixed Payment Loan**: The lender provides the borrower with an amount of funds, which must be repaid by making the same payment every period (i.e. month), consisting of part of the principal and interest for a set number of years.
  - E.g. auto loans and mortgages
Four Types of Credit Market Instruments cont’d

• **Coupon Bond**: pays the owner of the bond a fixed interest payment every year until the maturity date, when the final amount (par value or face value) is repaid.

• **Discount Bond**: (also called a zero-coupon bond) is a bond bought at a price lower than its face value with the face value repaid at the time of maturity. A discount bond does not make any interest payments.
  - E.g. Treasury bills
Yield to Maturity

• The interest rate that equates the present value of cash flow payments received from a debt instrument with its value today
Simple Loan—Yield to Maturity

PV = amount borrowed = $100
CF = cash flow in one year = $110

\[ n = \text{number of years} = 1 \]

\[
$100 = \frac{$110}{(1 + i)^1}
\]

\[ (1 + i) \times $100 = $110 \]

\[ (1 + i) = \frac{$110}{$100} \]

\[ i = 0.10 = 10\% \]

For simple loans, the simple interest rate equals the yield to maturity
• Or

\[ \$100 = \frac{\$110}{1+r} \]

Solving for \( i \),

\[ i = \frac{\$110 - \$100}{\$100} = \frac{\$10}{\$100} = 0.10 \text{ or } 10\% \]
Fixed Payment Loan—Yield to Maturity

The same cash flow payment every period throughout the life of the loan

LV = loan value
FP = fixed yearly payment
n = number of years until maturity

\[ LV = \frac{FP}{1+i} + \frac{FP}{(1+i)^2} + \frac{FP}{(1+i)^3} + \ldots + \frac{FP}{(1+i)^n} \]
Coupon Bond—Yield to Maturity

Using the same strategy used for the fixed-payment loan:

\[ P = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \ldots + \frac{C}{(1+i)^n} + \frac{F}{(1+i)^n} \]

- \( P \) = price of coupon bond
- \( C \) = yearly coupon payment
- \( F \) = face value of the bond
- \( n \) = years to maturity date
Example: Suppose your bond is selling for $950, and has a coupon rate of 7%; it matures in 4 years, and the par value is $1000. What is the YTM? The coupon payment is $70 (that's 7% of $1000), so the equation to satisfy is

\[
70/(1+r)^1 + 70/(1+r)^2 + 70/(1+r)^3 + 70/(1+r)^4 + 1000/(1+r)^4 = 950
\]
Consol or Perpetuity

- A bond with no maturity date that does not repay principal but pays fixed coupon payments forever

\[ P_c = \frac{C}{i_c} \]

- \( P_c \) = price of the consol
- \( C \) = yearly interest payment
- \( i_c \) = yield to maturity of the consol

Can rewrite above equation as \( i_c = \frac{C}{P_c} \)

For coupon bonds, this equation gives current yield—a easy-to-calculate approximation of yield to maturity
Discount Bond—Yield to Maturity

For any one year discount bond

\[ i = \frac{F - P}{P} \]

\( F = \) Face value of the discount bond
\( P = \) current price of the discount bond

The yield to maturity equals the increase in price over the year divided by the initial price.

As with a coupon bond, the yield to maturity is negatively related to the current bond price.
Other Measures of Interest Rates

- **current yield**
  - an approximation of the yield to maturity on coupon bonds that is often reported, because in contrast to the yield to maturity, it is easily calculated.
  - It is defined as the yearly coupon payment divided by the price of the security,
\[ I = \frac{C}{P} \]

Where

- \( i_c \) = current yield
- \( P \) = price of the coupon bond
- \( C \) = yearly coupon payment
On our one-year bill, which is selling for $900 and has a face value of $1,000, the yield on a discount basis would be as follows:

\[ i_{db} = \frac{\$1000 - \$900}{\$1000} \times \frac{360}{365} = 0.99 = 9.9\% \]
Rate of Return

The payments to the owner plus the change in value expressed as a fraction of the purchase price

\[
\text{RET} = \frac{C}{P_t} + \frac{P_{t+1} - P_t}{P_t}
\]

RET = return from holding the bond from time \( t \) to time \( t + 1 \)

\( P_t \) = price of bond at time \( t \)

\( P_{t+1} \) = price of the bond at time \( t + 1 \)

\( C \) = coupon payment

\[ \frac{C}{P_t} = \text{current yield} = i_c \]

\[ \frac{P_{t+1} - P_t}{P_t} = \text{rate of capital gain} = g \]
Rate of Return and Interest Rates

• The return equals the yield to maturity only if the holding period equals the time to maturity (time between when the bond is issued and when it matures)

• A rise in interest rates is associated with a fall in bond prices, resulting in a capital loss if time to maturity is longer than the holding period (expected period of time during which an investment is attributable to a particular investor)

• The more distant a bond’s maturity, the greater the size of the percentage price change associated with an interest-rate change
Rate of Return and Interest Rates (cont’d)

• The more distant a bond’s maturity, the lower the rate of return that occurs as a result of an increase in the interest rate

• Even if a bond has a substantial initial interest rate, its return can be negative if interest rates rise
<table>
<thead>
<tr>
<th>(1) Years to Maturity When Bond Is Purchased</th>
<th>(2) Initial Current Yield (%)</th>
<th>(3) Initial Price ($)</th>
<th>(4) Price Next Year* ($)</th>
<th>(5) Rate of Capital Gain (%)</th>
<th>(6) Rate of Return (2 + 5) (%)</th>
</tr>
</thead>
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<tr>
<td>30</td>
<td>10</td>
<td>1,000</td>
<td>503</td>
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<td>1</td>
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<td>1,000</td>
<td>1,000</td>
<td>0.0</td>
<td>+10.0</td>
</tr>
</tbody>
</table>

*Calculated using Equation 3.
Interest-Rate Risk

- The risk to which a portfolio is exposed to due to changes in the interest rates
- Prices and returns for long-term bonds are more volatile than those for shorter-term bonds
- There is no interest-rate risk for any bond whose time to maturity matches the holding period because the price at the end of the holding period is already fixed at the face value
  - The change in interest rates can then have no effect on the price at the end of the holding period for these bonds, and the return will therefore be equal to the yield to maturity known at the time the bond...
Real and Nominal Interest Rates

• Nominal interest rate makes no allowance for inflation
• Real interest rate is adjusted for changes in price level so it more accurately reflects the cost of borrowing
• Ex ante real interest rate is adjusted for expected changes in the price level
• Ex post real interest rate is adjusted for actual changes in the price level
Fisher Equation

- The Fisher equation states that the nominal interest rate $i$ equals the real interest rate $i_r$ plus the expected rate of inflation $\pi^e$:
  - $i = i_r + \pi^e$
- Rearranging terms, we find that the real interest rate equals the nominal interest rate minus the expected inflation rate:
  - $i_r = i - \pi^e$
• consider a situation in which you have made a one-year simple loan with a 5% interest rate \((i = 5\%)\) and you expect the price level to rise by 3\% over the course of the year \((\pi^e = 3\%)\).

• In this case, the interest rate you have earned in terms of real goods and services is 2\%; that is,

\[ \text{Ir} = 5\% - 3\% = 2\% \]
**Figure 1** Real and Nominal Interest Rates (Three-Month Treasury Bill), 1953–2005

Review Questions

- Would $100 tomorrow be worth more to you today when the interest rate is 20% or when it is 10%? Why
- If the interest rate is 10%, what is the PV of a security that pays you $1100 next year, $1210 the year after, and 1331 the year after that?
- What is the yield to maturity on a $1000 face value discount bond maturing in one year that sales for $800?
• **Question:** If the interest rate is 10%, what is the PV of a security that pays you $1100 next year, $1210 the year after, and $1331 the year after that?

• **Answer:** $\text{PV} = \frac{\text{CF}}{(1 + i)^n} = \frac{$1100}{(1 + 0.1)^1} + \frac{$1210}{(1 + 0.1)^2} + \frac{$1331}{(1 + 0.1)^3} = $1000 + $1000 + $1000 = $3000$
Review Questions & Answers Cont’d

• **Question:** What is the yield to maturity on a $1000 face value discount bond maturing in one year that sells for $800?

• **Answer:** For a discount bond, the yield to maturity \( (i) \) is calculated as follows:

\[
i = \frac{F - P}{P}, \text{ where } F = \text{ face value of a discount bond, } p = \text{ current price of a discount bond}
\]

\[
i = \frac{1000 - 800}{800} = \frac{200}{800} = \frac{1}{4} = 0.25 \text{ or } 25\%
\]
Question: Would $100 tomorrow be worth more to you today when the interest rate is 20% or when it is 10%? Why

Answer: $100 would be more worth when the interest rate is 10% because the lower the interest rate the higher the PV of $100 as shown below:

- When i = 20%, PV=100/(1+0.2) = $83.33
- When i = 10%, PV=100(1+0.1) = $90.91