Dear Students,

I trust that you are well and keeping safe. It is important for us all to keep complying with all protocols to minimize the infection rate and the negative impact of Covid-19.

Congratulations for completing and submitting your ITM111S assignment 2 despite the challenges of the present times.

We have tutor-marked and made comments in your answer scripts. Please take those comments seriously so that you can do better in your tests and other assessments.

There were common errors that we noticed in your work. To assist you in overcoming the challenges that majority of you faced with the assignment, here are some comments and hints question by question:

1.1.1 First list the union of B and C and then intersect this union with A.
1.1.2 First get the complement of A and then intersect this complement with B.
1.1.3 First get the intersection of A and B and then remove the elements of this intersection from C.

Read the given Venn diagram correctly.
1.2.1 Draw three interlocking circles in a box. Enter the given elements in their appropriate sections of your diagram. Label and indicate the values for each circle.

1.2.2/1.2.3/1.2.4 Use a correctly drawn Venn diagram to answer these questions.

2.1.1 This is addition of two matrices A and B.

2.1.2 Steps to getting inverse are obtain determinant, obtain Adjoint, and divide entries of Adjoint by the determinant.

2.1.3 Hint: Refer to how to find the product of two matrices as explained in pages 141 - 142 of your study guide.

2.2 Equate the determinant of the given matrix to zero. You will see that there is no valid equation to solve. Then you can pronounce yourself on the values of y as requested.

3.1 Start by clearing the fractions. To do this, multiply through by the LCM which is 4. Remember to draw the number line.

3.2.1 First term is 32 and the common difference is 4. Use the \( T_n \) formula for A.P

3.2.2 Use the \( S_n \) formula for A.P

3.3.1 Hint: \( ar^3 = r^3 \). This will give you the value of r. Then use the value of r to obtain the value of a.

3.3.2 Use the \( S_n \) formula for G.P

4.1 Think of how many men can dig 20m in 1 day. After that you should be able to answer the question.

4.2 Refer to pages 233 - 235 of your study guide.

4.3 Note that Ann’s investment is compound interest while Veronica’s is simple interest.

PLEASE BEAR IN MIND THAT ANSWERS DO NOT CARRY MARKS WHEN THERE ARE INCONSISTENCIES IN THE STEPS LEADING TO THEM.

We hope that you will use these comments and hints to improve on the subject in general.

Your marker-tutors for ITM111D wish you the very best.
Assignment 2 Memorandum

Question 1 (24 marks)

1.1 Consider the following Venn diagram:

List the elements of:

1.1.1 \( A \cap (B \cup C) \) 

\[ A \cap (B \cup C) = \{3, 9, 11, 14\} \]

1.1.2 \( A' \cap B \) 

\[ A' \cap B = \{2, 4, 5, 10\} \]

1.1.3 \( C - (A \cap B) \) 

\[ C - (A \cap B) = \{4, 5, 7, 8, 11, 16\} \]

1.2 In a school of 1800 students, 630 are girls \((G)\), 320 are students who can swim \((S)\) and 290 are students who are left-handed \((L)\). Furthermore, there are 80 girls who can swim, 30
left-handed students who can swim, 94 girls who are left-handed and 18 left-handed girls who can swim.

1.2.1 Present the given information on a Venn diagram as given below. (8)

1.2.2 How many Girls are right-handed and cannot swim at the school? (2)

\[ 630 - (62 + 18 + 76) = 474 \]

1.2.3 How many students are either left-handed or can swim? (2)

\[ 290 - (76 + 18 + 12) + 76 + 18 + 12 + 62 + 320 - (62 + 18 + 12) = 580 \]

1.2.4 How many girls cannot swim? (4)

\[ 474 + 76 = 550 \]
Question 2 (13)

2.1 Tobias sells two products, Televisions (T) and Video Cassette Recorders (V) in two shops, shop A and shop B. The numbers of products sold for the last two weeks in each shop are shown in the two matrices \( A \) and \( B \) as given below. The columns represent weeks and the rows correspond to products T and V respectively. Row one represents product T sold and row two represents product V sold.

\[
A = \begin{pmatrix} 3 & 4 \\ 5 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 4 & 1 \\ 5 & 0 \end{pmatrix}
\]

2.1.1 Derive a 2x2 matrix for the total of each product sold in both shops over the last two weeks.

\[
A + B = \begin{pmatrix} 7 & 5 \\ 10 & 2 \end{pmatrix}
\]

2.1.2 Find the inverse of matrix \( B \).

\[
det(B) = -5
\]

\[
B^{-1} = -\frac{1}{5} \begin{pmatrix} 0 & -1 \\ -5 & 4 \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{5} \\ 1 & -\frac{4}{5} \end{pmatrix}
\]

2.1.3 Determine the product \( AB \)

\[
AB = \begin{pmatrix} 32 & 3 \\ 30 & 5 \end{pmatrix}
\]
2.2 Determine the values of $y$ if the matrix $\begin{pmatrix} 2y^2 & 2 \\ 1 & 0 \end{pmatrix}$ has no inverse. (4)

$2y^2 \times 0 - 2 \times 1 = 0 \checkmark$

$-2 = 0 \checkmark$

This is not true under any condition. \checkmark

Hence the matrix has no inverse for any value of $y$.

**Question 3 (25)**

3.1 Solve the inequality below and illustrate your solution with a number line.

$-\frac{x}{2} + 5 - 3x \leq -8 - \frac{x}{4}$ (4)

$\times 4: -2x + 20 - 12x \leq -32 - x \checkmark$

$-13x \leq -52$

$x \geq 4 \checkmark$

$\bullet 4 \rightarrow$

3.2 The first three terms of an arithmetic progression are 32, 36, 40.

3.2.1 Determine the fiftieth (50th) term of the progression. (5)

$a = 32, \ d = 4 \checkmark$

$T_{50} = 32 + (50 - 1)4 \checkmark$

$= 228 \checkmark$

50th term is 228 \checkmark
3.2.2 Find the sum of the first twenty terms of the progression. (4)

\[ S_{20} = \frac{20}{2} \{64 + 19 \times 4\} \]
\[ = 1400 \]

Sum is 1400

3.3 The second term of a geometric progression is \(-81\) and the fifth term is 3.

3.3.1 What is the common ratio and what is the first term of the progression? (7)

\[ ar = -81 \quad ar^4 = 3. \]

\[ r^3 = -\frac{1}{27} \Rightarrow r = -\frac{1}{3} \]

\[ a = \frac{-81}{-\frac{1}{3}} = 243. \]

The common ratio is \(\frac{1}{3}\) and the first term is 243.

3.3.2 **Use** appropriate formula to determine the sum of the first six terms of the progression. (5)
\[ S_6 = \frac{243(1 - (-\frac{1}{3})^6)}{1 - -\frac{1}{3}} \]
\[ = \frac{243 \times \frac{728}{729}}{\frac{4}{3}} \]
\[ = 182 \]

**Sum is 182**

**Question 4 (16 marks)**

4.1 10 men can dig a 60-metre trench in 3 days. Working at the same rate, how many men can dig a 20-metre trench in 2 days? (4)

10 men dig 20m in 1 day ✓ ✓

∴ 5 men dig 20m in 2 days. ✓ ✓

4.2 The fuel capacities of a car, a bakkie and a truck are in the ratio 1.5:2:2.5. Share 1800 litres of fuel among these vehicles in the ratio of their capacities. (4)

*Total ratio is 6.* ✓

*Car:* \[ \frac{1.5}{6} \times 1800 = 450\text{litres} \] ✓

*Bakkie:* \[ \frac{2}{6} \times 1800 = 600\text{litres} \] ✓

*Truck:* \[ \frac{2.5}{6} \times 1800 = 750\text{litres} \] ✓

4.3 Ann invested N$15000 for 3 years at an annual interest rate of 12.5% compounded semi-annually. Veronica also invested N$15000 for 3 years at an annual simple interest rate of 12.5%. Use calculations to determine who made a wiser investment. (8)
Ann: \[ A = 15000 \left(1 + \frac{12.5}{2 \times 100}\right)^{2 \times 3} \]
\[ = 21580.67 \]

Veronica: \[ A = 15000 \left(1 + \frac{12.5}{100} \times 3\right) \]
\[ = 15000 \times 1.375 \]
\[ = 20625 \]

Ann made a wiser investment.