FEEDBACK TUTORIAL LETTER

1st SEMESTER 2019

ASSIGNMENT 2

INTRODUCTION TO MATHEMATICS

ITM111S
ASSIGNMENT 02 MEMORANDUM

1.1 Write the following numbers in descending order:

\[
\frac{33}{50} \quad 68.75\% \quad 0.673 \quad 2:3
\]

\[
68.75\% \quad 0.673 \quad 2:3 \quad \frac{33}{50}
\]

1.2 The ratio of men: women: children living in Outapi is 6:7:3; There are 42,000 women.

1.2.1 How many people altogether live in Outapi?

\[
\sqrt[7]{x} = 42000 \quad x = 96000 \quad \therefore 96000 \text{ people live in Outapi}
\]

1.2.2 How many children live in Outapi?

\[
\frac{3}{16} \times 96000 = 18000
\]

1.2.3 The 42,000 women is an increase of 20% on the number of women 10 years ago. Calculate how many women lived in Outapi ten years ago.

\[
x + 0.2x = 42000 \quad \text{or} \quad 42000 \times \frac{100}{120} = 35000
\]

\[
x = 35000 \quad \therefore \text{Number of women was 35000} \text{ 10 years ago}
\]

1.2.4 Twelve thousand of the children attend school and 48% of them are boys. Calculate the number of boys and the number of girls that attend school in Outapi.

\[
0.48 \times 12000 = 5760 \quad \text{boys}
\]

\[
12000 - 5760 = 6240 \quad \text{girls}
\]

2. It is given that \(-5 \leq x \leq -3\) and \(-1 \leq y \leq 2\). Find the largest possible value of:

\[
x = \{-5, -4, -3\} \text{ and } y = \{-1, 0, 1, 2\}
\]

2.1 \(x + y\)

\[
x + y = -3 + 2 = -1
\]
2.2 \[ xy \]
\[ xy = -5 \times 1 = 5 \]

2.3 \[ x^2y \]
\[ x^2y = (-5)^2 \times 2 = 50 \]

3.1 Use Cramer's rule to determine the solution of the following system of simultaneous equations:
\[ \frac{2x - 5y}{3} = 3 \quad \text{and} \quad \frac{7x}{3} = 5 + \frac{y}{3} \]
\[ 2x - 5y = 9 \]
\[ 7x - y = 15 \]

\[ D = \begin{vmatrix} 2 & -5 \\ 7 & -1 \end{vmatrix} = 33 \]
\[ D_x = \begin{vmatrix} 9 & -5 \\ 15 & -1 \end{vmatrix} = 66 \]
\[ D_y = \begin{vmatrix} 2 & 9 \\ 7 & 15 \end{vmatrix} = -33 \]
\[ x = \frac{66}{33} = 2 \quad \text{and} \quad y = \frac{-33}{33} = -1 \]

3.2 Given the following matrices:
\[ A = \begin{bmatrix} 2 & -4 \\ 1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 4 \\ 3 \end{bmatrix} \quad D = [3 \quad 1] \quad E = [\begin{bmatrix} -3 & 2 & 0 \\ 1 & -1 & -2 \end{bmatrix}] \]

Calculate, if possible:
3.2.1 \[ A + B \]
\[ A + B = \begin{bmatrix} 2 & -4 \\ 1 & 3 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & -3 \\ 0 & 5 \end{bmatrix} \]

3.2.2 \[ AC \]
\[ AC = \begin{bmatrix} 2 & -4 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} -4 \\ 13 \end{bmatrix} \]

3.2.3 \[ |B| \]
\[ |B| = (3 \times 2) - (1 \times -1) = 6 + 1 = 7 \]
3.2.4 \[ B^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} \]

3.2.5 \[ |A| = 10 \]

\[ A^{-1} = \frac{1}{10} \begin{bmatrix} 3 & 4 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{3}{10} & \frac{2}{5} \\ -\frac{1}{10} & \frac{1}{5} \end{bmatrix} \]

4. **2050** first year students were surveyed to determine their favorite cardiovascular equipment at the Student Recreation Center. The following results were obtained:

- **935** liked the treadmill machine
- **846** liked the elliptical machine
- **321** liked only the elliptical machine

4.1 Represent this information in a well labeled Venn diagram. Let \( T \) represent treadmill machine and let \( E \) represent the elliptical machine. [4]

4.2 Calculate what percentage of first years that liked both machines [2]
\[ n(T \cap E) = 525 \]
\[ \frac{525}{2050} \times 100 = 25.6\% \]

4.3 How many first years liked only the treadmill machine? [2]
\[ n(T - E) = 410 \]
\[ \div 410 \text{ students} \]

4.4 What percentage of students liked neither of the two machines? [2]
\[ n(T \cup E)^c = 794 \]
\[ \frac{794}{2050} \times 100 = 38.7\% \]
5.1 Shona invests $P$ for 4 months in an account with a simple interest rate of $6\%$ per year. If Shona receives $\$17$ as interest what amount $P$ did she invest? \[ I = Prt \]
\[ 17 = P \times 0.06 \times \frac{4}{12} \]
\[ P = \frac{17}{0.06 \times \frac{4}{12}} \]
\[ P = \$850 \]

5.2 Write the terms and determine the value of the sum \[ \sum_{i=3}^{6} (i+1)^2 \]
\[ \sum_{i=3}^{6} (i+1)^2 = (3+1)^2 + (4+1)^2 + (5+1)^2 + (6+1)^2 \]
\[ = 4^2 + 5^2 + 6^2 + 7^2 \]
\[ = 126 \]

6.1 Consider the progression 11, 8, 5, 2, .........
Determine whether the number $-150$ is a term of this progression? Justify by using mathematical rules
\[ T_n = a + (n-1)d \]
\[ -150 = 11 + (n-1)(-3) \]
\[ -150 = 11 - 3n + 3 \]
\[ 3n = 164 \]
\[ n = 54.67 \]

$-150$ is not a term in the AP as we get $n$ to be a fraction; $n$ can not be a fraction.
6.2 If \( A, B, C, \) and \( D \) are consecutive terms in an arithmetic progression, compute the value of \( \frac{D^2 - A^2}{C^2 - B^2} \). (take \( x \) to be your common difference) \[ \text{[10]} \]

\[ AP \rightarrow A, B, C, D \]

*but \( x \) is common difference*

\[ A, A + x, A + 2x, A + 3x \]

\[ D^2 - A^2 \]

\[ \frac{C^2 - B^2}{C^2 - B^2} \]

\[ = \frac{(A + 3x)^2 - A^2}{(A + 2x)^2 - (A + x)^2} \]

\[ = \frac{A^2 + 6Ax + 9x^2 - A^2}{A^2 + 4Ax + 4x^2 - (A^2 + 2Ax + x^2)} \]

\[ = \frac{6Ax + 9x^2}{2Ax + 3x^2} \]

\[ = \frac{3x(2A + 3x)}{x(2A + 3x)} = 3 \]

Total = 80