FEEDBACK TUTORIAL LETTER

1st SEMESTER 2019

ASSESSMENT 1

FOR

Applied Mathematical Economics
AME311S
1.1 USEFUL INFORMATION FOR WRITING YOUR COMING EXAM

Below are valuable information for improving and preparation for Exams:

- Please always attempt to all questions
- Never memorise the answer to the questions but rather learn the formulas and method of applications because questions might appear to be the same but the variables might have been changed.
- Be on a look out and vigilant about the sign changes as you move the variable to the other side of the equal sign.
- Always read the instruction and question carefully to understand and attempt as required.
- Always explain your steps and give indication of the rules and condition under which you are solving a problem.

1.2 ASSIGNMENT QUESTIONS

Instructions

- All questions are compulsory.
- In deciding how much detail to provide and how much time to spend on each question, it is imperative that you use the mark allocation as a guide.
- Show all your works and interpret your results.
- Use graphs wherever appropriate. Graphs must be clearly illustrated.

ASSIGNMENTS
Semester 1: 2019 Academic Session
Course Code: AME311S
Course Name: Applied Mathematical Economics

Assignment 1

Instructions

- Answer all questions
ASSIGNMENT 1

General Remarks

Only one student struggled with question 1 hence scored low marks. The average performance of most students is high in this Part of the Assignment. The correct answers are provided to assist the students when they revise for the upcoming examinations.

ASSIGNMENT 1

Question 1 [25 Marks]
Given $\alpha$ is non-income tax, $\beta$ is income tax, $\delta$ is marginal propensity to consume, $\gamma$ is autonomous consumption, $Y$ (national income), $I_0$ (investment) and $G_0$ (government expenditure)

1. Formulate the equations needed to find the reduced form of equilibrium income ($Y_e$).

\[
Y = \gamma + \delta(Y - T) + I_0 + G_0 \\
Y = \gamma + \delta Y - \delta T + I_0 + G_0 \\
Y = \gamma + \delta Y - \delta(\alpha + \beta Y) + I_0 + G_0 \\
Y = \gamma + \delta Y - \delta\alpha - \delta\beta Y + I_0 + G_0 \\
Y - \delta Y + \delta\beta Y = \gamma - \delta\alpha + I_0 + G_0 \\
(1 - \delta Y + \delta\beta)Y = \gamma - \delta\alpha + I_0 + G_0 \\
Y_e = \frac{\gamma - \delta\alpha + I_0 + G_0}{1 - \delta + \delta\beta} 
\]

(5)

2. Do a comparative static to find the effect of income tax, non-income tax and government spending on equilibrium income.

\[
\frac{\partial Y_e}{\partial \beta} = \frac{(1 - \delta Y + \delta\beta)(1 - \delta + \delta\beta)^2}{(1 - \delta + \delta\beta)(-\delta) - 0(y - \delta\alpha + I_0 + G_0)} = \frac{\delta(y - \delta\alpha + I_0 + G_0)}{(1 - \delta + \delta\beta)^2} = \frac{-\delta Y_e}{1 - \delta + \delta\beta} \\
\frac{\partial Y_e}{\partial \alpha} = \frac{1 - \delta}{1 - \delta + \delta\beta}(1 - \delta + \delta\beta)(-\delta) - 0(y - \delta\alpha + I_0 + G_0) = \frac{(1 - \delta + \delta\beta)^2}{(1 - \delta + \delta\beta).(-\delta)} = \frac{\delta}{1 - \delta + \delta\beta} \\
\frac{\partial Y_e}{\partial I_0} = \frac{(1 - \delta + \delta\beta)(1 - \delta + \delta\beta)0}{(1 - \delta + \delta\beta)^2} = \frac{(1 - \delta + \delta\beta)^2}{(1 - \delta + \delta\beta)^2} = \frac{1}{1 - \delta + \delta\beta} \\
(15)
\]

3. If $\beta = 0.2; \alpha = 20; \gamma = 80; \delta = 0.25; I_0 = 45; G_0 = 50$, find the effects of lump sum tax increase by $1$ billion?

\[
\frac{\partial Y_e}{\partial \alpha} = \frac{-\delta}{1 - \delta + \delta\beta} = \frac{-0.25}{1 - 0.25 + 0.25(0.2)(1000 000 000)} = -312 500 000 
\]

(5)
Question 2 [25 Marks]
Consider the following national income model (tax ignored).

\[ Y - C(Y) - I(i) - G_0 = 0 \quad [0 < C' < 1; I' < 0] \]
\[ kY + L(i) - M_{g0} = 0 \quad [k > 0; L' < 0] \]

Analyse the comparative statics of the model to find the effect of expansionary monetary and fiscal policy?

First set the implicit functions

\[ Y - C(Y) - i(i) - G_0 = 0 \]
\[ kY + L(i) - M_{g0} = 0 \]

The basic rule

\[ dF^1 = \frac{\partial F^1}{\partial y} dy + \frac{\partial F^1}{\partial i} di + \frac{\partial F^1}{\partial G_0} dG_0 = 0 \]
\[ dF^2 = \frac{\partial F^2}{\partial y} dy + \frac{\partial F^2}{\partial i} di + \frac{\partial F^2}{\partial M_{g0}} dM_{g0} = 0 \]

\[ = (1 - C_y)dy + (-I')di + (-1)dG_0 = 0 \quad (1) \]
\[ = kdy + L_idi + (-1)dM_{g0} = 0 \quad (2) \]

Arranging (1) and (2) gives

\[ = (1 - C_y)dy - I'di = dG_0 \quad (3) \]
\[ = kdy + L_idi = dM_{g0} \quad (4) \]

To see the effect of fiscal policy, set \( dM_{g0} = 0 \) and divide (3) and (4) by \( dG_0 \) to get;

\[ (1 - C_y) \frac{\partial y}{\partial G_0} - I' \frac{\partial i}{\partial G_0} = 1 \]
\[ k \frac{\partial y}{\partial G_0} + L_i \frac{\partial i}{\partial G_0} = 0 \]

In JX=B form:

\[
\begin{bmatrix}
1 - C_y & -I' \\
k & L_i
\end{bmatrix}
\begin{bmatrix}
\frac{\partial y}{\partial G_0} \\
\frac{\partial i}{\partial G_0}
\end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}
\]

We find the Jacobian determinant \( |J| \),

\[ |J| = \begin{vmatrix}
1 - C_y & -I' \\
k & L_i
\end{vmatrix} = (1 - C_y)L_i + kI' = (+)(-) + (+)(-) = (-) \text{ or } |J| < 0 \]

Using Cramer's rule, we form a new form matrix, \( |J| \) to get the solution,
If \( G_0 \) increases by N\$1, then \( y \) will increase by \( \frac{L_i}{1 - C_y L_i + k l_i'} \).

\[
\frac{\partial y}{\partial G_0} = \left| \begin{array}{cc} 1 & -l_i' \\ 0 & L_i \end{array} \right| = \frac{L_i}{(1 - C_y L_i + k l_i')} = \left( \frac{(-)}{(-)} \right) > 0
\]

If \( G_0 \) increases by N\$1, then \( i \) will increase by \( \frac{-k}{1 - C_y L_i + k l_i'} \).

\[
\frac{\partial i}{\partial G_0} = \left| \begin{array}{cc} 1 - C_y & 1 \\ k & 0 \end{array} \right| = \frac{-k}{(1 - C_y L_i + k l_i')} = \left( \frac{(-)}{(-)} \right) > 0
\]

To see the effect of monetary policy, set \( dM_0 = 0 \) and divide (3) and (4) by \( dG_0 \) to get:

\[
(1 - C_y) \frac{\partial y}{\partial M_0} - l_i' \frac{\partial i}{\partial M_0} = 0
\]

\[
k \frac{\partial y}{\partial M_0} + L_i \frac{\partial i}{\partial M_0} = 1
\]

In JX=B form:

\[
\begin{bmatrix}
1 - C_y & -l_i' \\
k & L_i
\end{bmatrix}
\begin{bmatrix}
\frac{\partial y}{\partial M_0} \\
\frac{\partial i}{\partial M_0}
\end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}
\]

Using Cramer’s rule, we form a new form matrix, \( |J_1| \) to get the solution,

\[
\frac{\partial y}{\partial M_0} = \frac{|J_1|}{|J|} = \frac{0}{1} = \frac{-l_i'}{(1 - C_y L_i + k l_i')} = \left( \frac{(-)}{(-)} \right) > 0
\]

If \( M_0 \) increases by N\$1, then \( y \) will increase by \( \frac{l_i'}{(1 - C_y L_i + k l_i')}. \)

\[
\frac{\partial i}{\partial M_0} = \frac{|J_2|}{|J|} = \frac{1 - C_y}{1} = \frac{1 - C_y}{(1 - C_y L_i + k l_i')} = \left( \frac{(+)}{(-)} \right) < 0
\]

If \( M_0 \) increases by N\$1, then \( i \) will decrease by \( \frac{1 - C_y}{(1 - C_y L_i + k l_i')}. \)

**Question 3 [25 Marks]**

1. Use Jacobian determinants to test the existence of functional dependence between the paired functions.
   a) 
   
   \[
   \begin{align*}
   y_1 &= 3x_1^2 + x_2 \\
y_2 &= 9x_1^4 + 6x_1^2(x_2 + 4) + x_2(x_2 + 8) + 12 \\
y_3 &= 9x_1^4 + 6x_1^2x_2 + 24x_2^2 + x_2^2 + 8x_2 + 12 \\
y_{11} &= 6x_1 \\
y_{12} &= 1 \\
y_{21} &= 36x_1^3 + 12x_1x_2 + 48x_1 \\
y_{22} &= 6x_1^2 + 2x_2 + 8
   \end{align*}
   \]
Thus, since $|J| = 0$, there is functional dependence.

\[ |J| = 6x_1 \left(6x_1^2 + 2x_2 + 8\right) - \left[36x_1^3 + 12x_1x_2 + 48x_1\right] \]

\[ 36x_1^3 + 12x_1x_2 + 48x_1 - 36x_1^2 - 12x_1x_2 - 48x_1 = 0 \]

Since $|J| = 0$, there is functional dependence.

b) 
\[
\begin{align*}
y_1 &= 3x_1^2 + 2x_2^2 \\
y_2 &= 5x_1 + 1 \\
y_{11} &= 6x_1 \\
y_{12} &= 4x_2 \\
y_{21} &= 5 \\
y_{22} &= 0
\end{align*}
\]

Thus, 
\[
|J| = \begin{vmatrix} 6x_1 & 4x_2 \\ 5 & 0 \end{vmatrix} = 6x_1(0) - 5(4x_2) = -20x_2
\]

Since $|J| \neq 0$, there is functional dependence.

2. Optimise the following function, using a) Cramer’s rule for the first order condition and b) the Hessian for the second-order condition:

\[ y = 5x_1^2 - 7x_1 - x_1x_2 + 8x_2^2 - 6x_2 + 4x_2x_3 + 6x_3^2 + 4x_3 - 5x_1x_3 \]

a) The first-order conditions are
\[
\begin{align*}
y_1 &= 10x_1 - 7 - x_2 - 5x_3 = 0 \\
y_2 &= -x_1 + 16x_2 - 6 + 4x_3 = 0 \\
y_3 &= 4x_2 + 12x_3 + 4 - 5x_1 = 0
\end{align*}
\]

which in matrix form is
\[
\begin{bmatrix} 10 & -1 & -5 \\ -1 & 16 & 4 \\ -5 & 4 & 12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \\ -4 \end{bmatrix}
\]

Using Cramer’s rule, $|A| = 10(176) + 1(8) - 5(76) = 1388$, since $|A|$ also equals $|J|$, the equations are functionally independent.

\[ |A_1| = 7(176) + 1(88) - 5(88) = 880 \]
Thus,

\[ |A_2| = 10(88) - 7(8) - 5(34) = 654 \]

\[ |A_3| = 10(-88) + 1(34) + 7(76) = -314 \]

Thus,

\[ \tilde{x}_1 = \frac{880}{1388} = 0.63 \quad \tilde{x}_2 = \frac{654}{1388} = 0.47 \quad \tilde{x}_3 = \frac{-314}{1388} = 0.23 \]

b) Testing the second-order condition by taking the second-order partials to form the Hessian,

\[ y_{11} = 10 \quad y_{12} = -1 \quad y_{13} = -5 \]

\[ y_{21} = -1 \quad y_{22} = 16 \quad y_{23} = 4 \]

\[ y_{31} = -5 \quad y_{32} = 4 \quad y_{33} = 12 \]

Thus,

\[ |H| = \begin{vmatrix} 10 & -1 & -5 \\ -1 & 16 & 4 \\ -5 & 4 & 12 \end{vmatrix} \]

Where,

\[ |H_1| = 10 > 0 \quad |H_2| = \begin{vmatrix} 10 & -1 \\ -1 & 16 \end{vmatrix} = 159 > 0 \quad \text{and} \quad |H_3| = 1388 > 0 \]

Hence \(|H_1| > 0, |H_2| > 0\) and \(|H_3| > 0, |H|\) positive definite, and \(y\) is minimized at the critical values \((15)\)

Question 4 [25 Marks]

1. Give the input matrix and the final demand vector

\[ A = \begin{bmatrix} 0.05 & 0.25 & 0.34 \\ 0.33 & 0.10 & 0.12 \\ 0.19 & 0.38 & 0 \end{bmatrix} \quad d = \begin{bmatrix} 800 \\ 400 \\ 700 \end{bmatrix} \]

(a) Find the solution output levels by Cramer’s rule

From the above matrix, let \((I - A) = V\) so that our matrix represent \(Vx = d\), then we can establish the determinant and form the special matrix to allow us to use Cramer’s rule

\[ V = \begin{bmatrix} 0.95 & -0.25 & -0.34 \\ -0.33 & 0.90 & -0.12 \\ -0.19 & -0.38 & 1 \end{bmatrix} \]

\[ |V| = 0.95 \begin{vmatrix} 0.90 & -0.12 \\ -0.33 & -0.19 \end{vmatrix} + 0.25 \begin{vmatrix} -0.33 & -0.12 \\ -0.19 & 1 \end{vmatrix} - 0.34 \begin{vmatrix} -0.33 & 0.90 \\ -0.19 & -0.38 \end{vmatrix} = 0.6227 \]

\[ |V_1| = \begin{vmatrix} 800 & -0.25 & -0.34 \\ 400 & 0.90 & -0.12 \\ 700 & -0.38 & 1 \end{vmatrix} \]
Find the new output level when final demands increase by 10%, 40%, and 20% respectively.

New final demands \( d = \begin{bmatrix} 880 \\ 560 \\ 840 \end{bmatrix} \) our \( V \) remains the same hence \( |V| = 0.6227 \)

\[
\begin{align*}
|V_1| &= \begin{vmatrix} 880 & -0.25 & -0.34 \\ 560 & 0.90 & -0.12 \\ 840 & -0.38 & 1 \end{vmatrix} \\
&= 880 \begin{vmatrix} 0.90 & -0.12 \\ -0.38 & 1 \end{vmatrix} + 0.25 \begin{vmatrix} 560 & -0.12 \\ -0.38 & 1 \end{vmatrix} - 0.34 \begin{vmatrix} 560 & 0.90 \\ 840 & -0.38 \end{vmatrix} = 1246.464 \\
|V_2| &= -0.33 \begin{vmatrix} 560 & -0.12 \\ 840 & 1 \end{vmatrix} - 0.19 \begin{vmatrix} 560 & -0.33 \\ 840 & 1 \end{vmatrix} - 0.34 \begin{vmatrix} 560 & 0.90 \\ 840 & -0.25 \end{vmatrix} = 375.368 \\
|V_3| &= -0.33 \begin{vmatrix} 560 & -0.12 \\ 840 & 1 \end{vmatrix} - 0.19 \begin{vmatrix} 560 & -0.33 \\ 840 & 1 \end{vmatrix} - 0.34 \begin{vmatrix} 560 & 0.90 \\ 840 & -0.25 \end{vmatrix} = 1138.492 \\
\end{align*}
\]

\[
\begin{align*}
x_1^* &= \frac{|V_1|}{|V|} = \frac{1246.464}{0.6227} = 2001.71 \\
x_2^* &= \frac{|V_2|}{|V|} = \frac{375.368}{0.6227} = 602.81 \\
x_3^* &= \frac{|V_3|}{|V|} = \frac{1138.492}{0.6227} = 1828.32 \\
\end{align*}
\]
In a three–industry economy, it is known that industry I uses 20 cents of its own product, 10 cents of commodity III and 60 cents of commodity II to produce a dollar's worth of commodity I industry II uses 10 cents of its own product, 30 cents of commodity III and 50 cents of commodity I to produce a dollar's worth of commodity II while industry III uses none of its own product and commodity I, but uses 20 cents of commodity II in producing a dollar’s worth of commodity III; and the open sector demands N$ 2,000 billion of commodity I, N$ 500 billion of commodity II and 1500 billion of commodity III.

a) Write out the input matrix, and the specific input matrix equation for this economy.

\[
\begin{bmatrix}
0.2 & 0.5 & 0 \\
0.6 & 0.1 & 0.2 \\
0.1 & 0.3 & 0
\end{bmatrix}
\begin{bmatrix}
X_1 \\
X_2 \\
X_3
\end{bmatrix}
= 
\begin{bmatrix}
2000 \\
500 \\
1500
\end{bmatrix}
\]

b) Find the solution output levels by Cramer's rule.

From the above matrix, let \((I - A) = V\) so that our matrix represent \(VX = d\), then we can establish the determinant and form the special matrix to allow us to use Cramer’s rule.

\[
|V| = 0.8(0.84) + 0.5(-0.58) = 0.362
\]

\[
|V_1| = \begin{vmatrix}
2000 & -0.5 & 0 \\
500 & 0.9 & -0.2 \\
1500 & -0.3 & 1
\end{vmatrix} = 1000
\]

\[
|V_2| = \begin{vmatrix}
0.8 & 2000 & 0 \\
-0.6 & 500 & -0.2 \\
-0.1 & 1500 & 1
\end{vmatrix} = 2240
\]

\[
|V_3| = \begin{vmatrix}
0.8 & -0.5 & 2000 \\
-0.6 & 0.9 & 500 \\
-0.1 & -0.3 & 1500
\end{vmatrix} = 1315
\]

Then our \(x_1, x_2, \text{and} x_3\)

\[
x_1 = \frac{|V_1|}{|V|} = \frac{1000}{0.362} = 2762.43
\]

\[
x_2 = \frac{|V_2|}{|V|} = \frac{2240}{0.362} = 6187.85
\]

\[
x_3 = \frac{|V_3|}{|V|} = \frac{1315}{0.362} = 3632.7
\]

TOTAL MARKS: 100
1.3 PERFORMANCE STATISTICS FOR ASSIGNMENT 1/2018

Total number of students who submitted Assignment 1 = 4
Total score = 289
Average score = 72%
Students who scored above 50% = 3 (75%)
Borderline cases, that is, 50% = 0 (0%)
Students who scored below 50% = 1 (25%)
Highest score = 88%
Lowest score = 46%

1.4 EXAMINATION: JUNE 2019

The First Opportunity Examination for June 2019 (as well as the Supplementary Examination taking place in July 2019) will cover all topics except of this course as suggested by the course outline and the current study guide. These are:

- Review of Mathematics for comparative static analysis;
- Cramer’s rule,
- Special types of matrices:
  - Jacobean Hessian and Bordered Hessian;
  - Sign definiteness of quadratic forms.
- Input-Output analysis (static models).
- Linear programming,
- Non linear programming
- Introduction to dynamic analysis
- First order differential and difference equation
- Review of Mathematics for comparative static analysis;
- Cramer’s rule,
- Special types of matrices:
  - Jacobean Hessian and Bordered Hessian;
  - Sign definiteness of quadratic forms.
- Input-Output analysis (static models).
- Linear programming.
Furthermore, you are strongly advised to study the **assignment questions and all the test questions** for an insight into the standard of questions that may be set around the above-identified issues. This questions papers and their memo are available on request to my email address eshipanga@nust.na I will reply to make sure I forward you the soft copies electronically.

Also, do consider the formation of **Study Groups** in order to share and cross-fertilise ideas on issues related to the course. Furthermore, you should keep in close **contact with the full-time and part-time students** since they are benefiting a great deal from the daily face-to-face contacts with their Lecturer. Do not forget to read your Study Guide for the course. Finally, do not hesitate to contact Mr. Eden Tate Shipanga (The Lecturer for this course) should the need arise.

### 1.5 ACTIVITIES IN THE STUDY GUIDE

The activities in the study guide are meant to provide additional intellectual challenge to you. Do attempt to go through the activities in the study guide as they will really provide you with practice questions needed for the mastery of the content for this course.

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Eden Tate Shipanga  
March 2018